Assignment 6 - Metric Spaces

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1. Check whether the following are metric spaces.

   (i) $(\mathbb{R}^2, d)$ where $d(x,y) := |x_1 - y_1| + |x_2 - y_2|$.

   (ii) $(\mathbb{R}^2, d)$ where $d(x,y) := (x_1 - y_1)^2 + (x_2 - y_2)^2$.

   (iii) (X, d') where given a metric space (X, d) we define
   $$d'(x,y) := \mathsf{min}\big\{1, d(x,y)\big\}.$$

   (iv) $(\mathbb{R}^2, d)$ where $d(x,y) = |x_1 - y_1|$.

   (v) (X, d) where X is any set and $d(x,y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y. \end{cases}$.

2. Describe open balls for the following metric spaces.

   (i) $(\mathbb{R}^2, d)$ where $d(x,y) := |x_1 - y_1| + |x_2 - y_2|$.

   (ii) $(\mathbb{R}^2, d)$ where $d(x,y) := \mathsf{min}\big\{1, \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}\big\}$.

   (iii) $(\mathbb{R}^2, d)$ where $d(x,y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y. \end{cases}$.

3. In $\mathbb{R}^2$ with $d(x,y) := \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ show that a sequence $x_n = (x_{n,1}, x_{n,2})$ converges to a point $p = (p_1, p_2)$ if and only if $x_{n,1}$ converges to p_1 and and $x_{n,2}$ converges to p_2.

4. What are the convergent sequences in (X, d) where d is the discrete metric on X as in Question 1(v)?