

Assignment 5 - Normal subgroups, quotient groups - II

- 1 For a subgroup H of a group G , define index of H in G to be the number of left cosets of H . Show that an index two subgroup of any group is always normal.
- 2 (First isomorphism theorem) : Let $\phi : G \rightarrow G'$ be a homomorphism. Show that $G/\text{Ker}(\phi)$ is isomorphic to $\text{Im}(\phi)$.
- 3 Let M, N be normal subgroups of G . Show that the set $MN = \{mn \mid m \in M, n \in N\}$ is a normal subgroup of G .
- 4 Give an example of a non-normal subgroup of $GL_n(\mathbb{R})$.
- 5 Let $SL_n(\mathbb{R})$ is normal in $GL_n(\mathbb{R})$. Show that $GL_n(\mathbb{R})/SL_n(\mathbb{R}) \cong \mathbb{R}^\times$.¹

¹the expression $G \cong G'$ means G is isomorphic to G' .