

# Assignment 4 - Normal subgroups, quotient groups

- 1 If  $G$  is a finite group, show that for any  $a \in G$ ,  $a^{|G|} = e$ .
- 2 Let  $G$  be a group with  $|G| = p$  where  $p$  is a prime number. Show that  $G$  is cyclic.
- 3 Let  $p$  be a prime number and  $a$  an integer. Show that  $a^p \equiv a \pmod{p}$
- 4 Show that every subgroup of an abelian group is normal.
- 5 Show that every normal subgroup of  $G$  is kernel of some homomorphism from  $G$ .