

Assignment 3 - Homomorphisms and cosets

- 1 Define (i) homomorphism of groups (ii) left coset of a subgroup.
- 2 Which of the following are group homomorphisms? (Explain)
 - 1 The map $\phi : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $x \mapsto x + 1$.
 - 2 The map $\phi : G \rightarrow G$ given by $x \mapsto x^{-1}$.
 - 3 For G abelian, the map $\phi : G \rightarrow G$ given by $x \mapsto x^{2017}$.
 - 4 $\phi : \mathbb{R}^\times \rightarrow \mathbb{R}^\times$ given by $x \mapsto e^x$.
- 3 For a fixed element $x \in G$, the map $\phi : G \rightarrow G$ defined by $y \mapsto xyx^{-1}$. Show that it is a bijective homomorphism (bijective homomorphisms are called isomorphisms).
- 4 Let $\phi : G \rightarrow G'$ be a homomorphism. Show that $\phi(G)$ (the image of ϕ) is a subgroup of G' .
- 5 Show that a homomorphism ϕ is injective iff $\text{Ker}(\phi)$ is trivial.
- 6 For a left coset aH of H , show that the map $H \rightarrow aH$ given by $h \mapsto ah$ is a bijection.
- 7 Show that any two left cosets of a subgroup are either disjoint or equal.