Assignment 12 - Inner Product spaces

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In all the questions below, let Let V be a vector space with an inner product denoted by $< \, , \, >$.

1. Show that for $v, w \in V$,
$$| < v, w > | \leq ||v|| \cdot ||w||.$$
(This inequality is called Cauchy Schwartz inequality.)

2. Let $\widetilde{V}$ denote the completion of V. For elements $\tilde{v}, \tilde{w} \in \widetilde{V}$ represented by Cauchy sequences in (v_n) and (w_n) respectively, show that defining
$$< \tilde{v}, \tilde{w} >:= \varinjlim_{n \to \infty} < v_n, w_n >$$
gives a well defined inner product on the vector space $\widetilde{V}$.

3. Now let $< \, , \, >$ be an inner product on $\mathbb{R}^n$. Let $\{e_1, ..., e_n\}$ be an orthonormal basis of $\mathbb{R}^n$. Show that for any $v \in \mathbb{R}^n$,
$$v =< v, e_1 > e_1 + \cdots + < v, e_n > e_n.$$