Assignemnt 6

1. Let f be continuous at a and $f(a) > 0$. Show $\exists\ \delta > 0$ such that $f(x) > 0$ for all x satisfying $|x - a| < \delta$. In other words, if a continuous function is positive at a point, it remains positive at nearby points.
2. Define $f'(a)$, the derivative of a function f at a. Give examples to show that derivative may not exist.
3. Prove that the function f defined by $f(x) = |x|$ is a continuous but not differentiable at 0.
4. Calculate $\lim_{x \downarrow 0} \frac{sin|x|}{x}$ and $\lim_{x \uparrow 0} \frac{sin|x|}{x}$
5. Show that if g is continuous at a and f is continuous at $g(a)$, then $f \circ g$ is continuous at a.