Assignemnt 5

1. Assuming triangle inequality (i.e. $|x+y| \leq |x| + |y|$) show that for any two real numbers x, y

$$|x| - |y| \leq |x - y|$$

   Give examples to show that the inequality can be strict.

2. $\lim_{x \to a} f(x) = \ell$ and $\lim_{x \to a} g(x) = m$. Show that

$$\lim_{x \to a} \Big(f(x) \cdot g(x) \Big) = \ell \cdot m$$

3. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$ such that $\lim_{x \to a} f(x)$ exists but is not equal to $f(a)$.

4. Show

$$\lim_{x \to 1} \frac{x^5 + 27x^2 + 3}{x^{10} + 1} = \frac{35}{11}$$

   You may use theorems discussed in class, provided you clearly state them.