Assignemnt 2

1. Define 'bounded sequence'. Prove that a convergent sequence is bounded.
2. Suppose $\{x_n\}$ and $\{y_n\}$ are convergent sequences converging to x and y respectively. Prove that
   - $\lim_{n\to\infty}(x_n - y_n) = x - y$
   - $\lim_{n\to\infty}(x_n y_n) = xy$
   - For any real number c, $\lim_{n\to\infty}(cx_n) = cx$
3. Show that the sequence $x_n = \sqrt{n}$ does not converge to any real number.
4. What do you mean by a Cauchy sequence?
5. State and prove triangle inequality for absolute value of real numbers.
6. Let x_n be the sequence defined by

$$x_n = \text{number of distinct prime factors of } n$$

For e.g. $x_1 = 0$, $x_2 = 1$, $x_3 = 1$, $x_6 = 2$ and so on. Is x_n convergent? explain.