ASSIGNMENT 10

MTH101 (2014)

(1) State l'Hôpital's rule. Find $\lim\limits_{x \to 0} \frac{x}{tan\ x}$ using this rule.

(2) What is wrong with the following use of l'Hôpital's rule:
$$\lim_{x \to 1} \frac{x^3 + x - 2}{x^2 - 3x + 2} = \lim_{x \to 1} \frac{3x^2 + 1}{2x - 3} = \lim_{x \to 1} \frac{6x}{2} = 3.$$
(The limit is actually -4!).

(3) Prove that if
$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0$$
then there exists an $x \in [0, 1]$ such that
$$a_0 + a_1 x + ... + a_n x^n = 0$$

(4) Calculate the critical points of the function $f(x) = x + \frac{3}{x^2}$. Can the second derivative f'' be used to say determine whether these critical points are local maxima or minima? Sketch the graph of this function.