1. Consider a system of N free particles in which the energy of each particle can assume two distinct values, 0 and E. Let n_0 particles be at 0-energy state and N-n_0 particles are in the E-energy state. The total energy of the system is U.
   a. Find the entropy of such a system.
   b. Find the most probable values of number of particles in each state. and find the mean square fluctuation of these quantities.
   c. Find the temperature of the system.
   (problem taken from Huang, can consider the reference therein to clarify for the possible negative temperature of the system.)

2. A simple harmonic oscillator has energy states $E_n = (n+1/2)h\omega$, where h and ω are constants, $n = 0, 1, 2...$ all integers. The system is at a temperature T.
   a. Find the ration of probability of the oscillator being in the first excited state to that being in the ground state.
   b. Assuming that the ground and the first excited states are only occupied find the mean energy of the oscillator.
   c. Find an expression of the mean energy of the oscillator when all the states are occupied according to the Boltzmann distribution. (Reif, chap. 6, Q.1)

3. Consider N weakly interacting spin 1/2 and magnetic moment μ particles in an external magnetic field H. Let the system be at a temperature T. Calculate the mean energy $< E >$ and magnetic monent of the system. (Reif, chap 6, Q.2)

4. A system consistes N weakly interacting particles, each of which can be in one of the two states of energy ϵ_1 and ϵ_2 ($\epsilon_1 < \epsilon_2$.
   a. Without explicit calculations show How the average energy of the system should change with temperature.
   b. Using the above plot show how C_V changes with temperature.
   c. Calculate explicitly $< E >$ and C_V and compare with the above qualitative plots. (Reif, chap 6, Q.6)

5. A solid at absolute temperature T contains N negatively charged impurity ions per cubic centimetre, these ions replace some ordinary atoms of the solid. Each negative ion of charge -e has in its vicinity mobile positive ions which can sit at the middle of the square lattice where at each lattice point there is a negative ion (see the schematic diagram in Reif's book, chap 6, Q.8). If a small electric field ϵ is applied in the x-direction, calculate the mean electric dipole moment per unit volume.

6. A dilute solution of macro-molecules at temperature T is placed in a centrifuge rotating with angular velocity ω. Considering a molecule of mass m experiences a radially outward force $m\omega^2 r$ at a distance r from the axis;
   a. Find how the relative density $\rho(r)$ of molecules varies with their radial distance.
   b. Show quantitatively how the molecular weight os the macromolecule can be determined if the density ratio ρ_1/ρ_2 at the redii r_1 and r_2. (Reif, Q. 6.10)

7. Consider a rectangular box with four walls and a bottom but no top. The total area of the sides and the bottom is A. Find the dimensions of the box which gives a maximum volume using
   a. the method of straight forward calculus,
   b. Lagrange multipliers. (Reif, Q. 6.12)

8. An ideal monatomic gas of N particles, each of mass m, is in thermal equilibrium at a temperature T. The gas is contained in a cubical box of sides L, whose top and bottom surfaces are parallel to the ground. Considering the presence of the gravitational acceleration,
   a. find the average K.E. of the particles,

b. and find the average potential energy of the particles. (Reif, Q. 7.2)

9. N weakly interacting particles at a high temperature T undergo one dimensional oscillations about some mean positions. calculate the heat capacity of
a. when the restoring force causing the oscillation is proportional to displacement x from mean position,
b. when the restoring force is proportional to x^3. (Reif, Q. 7.10)

10. An aqueous solution at room temperature T contains the small concentration of magnetic atoms, each of which has a net spin 1/2 and a magnetic moment μ. The solution is placed in an external magnetic field H pointing along z-direction. The field is inhomogeneous $H = H(z)$ where $H(Z)$ is a monotonically increasing function of Z. Consider H at $Z = Z_1(bottom)$ is H_1 and at $Z = Z_2(top)$ is H_2.
a. Let $n_+(z)dz$ be the mean number of atoms whose spin points along the z direction at the height z and z+dz. What is the ration $n_+(z_2)/n_+(Z_1)$.
b. Let $n(z)dz$ be the total number of magnetic atoms (both up and down) at a height z. What is the ratio $n(z_2)/n(z_1)$?
c. Considering $\mu H << kT$, simplify the above expressions. (Reif, Q. 7.16)

11. an ideal monatomic gas is in thermal equilibrium at temperature T (velocity distribution Maxwellian).
a. If v is the speed of molecules, calculate $\bar{1/v}$ and $1/\bar{v}$.
b. find the mean number of molecules at an energy ϵ and $\epsilon + d\epsilon$ (Reif, Q. 7.20)