QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT V

(1) What is the conjugate momenta corresponding to ψ ? What is the Hamiltonian. Express the Hamiltonian in terms of the Fock space operators.

(2) Derive an expression for $\langle 0|T\{\psi(x)\bar{\psi}(y)\}|0\rangle$. What is this object ? What happens if you choose the wrong commutation relation ?

(3) Starting from the general solution of the Dirac equation, show that

$$
\begin{aligned}
u_s(p) &= -i\gamma^2(v_s(p))^* \\
v_s(p) &= -i\gamma^2(u_s(p))^*
\end{aligned}
$$

[Hint: $\xi^{-s} = -i\sigma_2(\xi^s)^*$]

(4) Show the following

$$
\begin{aligned}
(\gamma^0)^\dagger &= \gamma^0 \\
(\gamma^k)^\dagger &= -\gamma^k \\
\gamma^0(\gamma^\mu)^\dagger\gamma^0 &= \gamma^\mu
\end{aligned}
$$

(5) What are global and local symmetry transformations?

(6) How is minimal coupling of photons to fermions incorporated ? How is this related to the idea of gauge symmetries?

(7) Show that

$$
\slashed{D}^2 = \mathcal{D}^2 + \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}
$$

What is the new equation of motion ?

Extra : Attempt problem (10.1) from the textbook *Quantum Field Theory and the Standard Model, Matthew D. Schwartz.*