QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT IV

(1) Solve the Dirac equation in the rest frame and in a frame boosted along the Z-axis.

(2) Show that

$$\begin{aligned} \bar{u}_s(p)u_{s'}(p) &= 2m\delta_{ss'} \\ \bar{v}_s(p)v_{s'}(p) &= -2m\delta_{ss'} \end{aligned}$$

(3) Show that the spinor outer products satisfy

$$\begin{aligned} \sum_{s=1}^{2} u_s(p)\bar{u}_s(p) &= \not{p} + m \\ \sum_{s=1}^{2} v_s(p)\bar{v}_s(p) &= \not{p} - m \end{aligned}$$

(4) What are the fourier mode expansions for ψ and $\bar{\psi}$? Postulating the equal-time anticommutation relations

$$\begin{aligned} \{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} &= \delta_{ab}\ \delta^{(3)}(\vec{x}-\vec{y}) \\ \{\psi_a(\vec{x}), \psi_b^(\vec{y})\} &= \{\psi_a^\dagger(\vec{x}), \psi_b^\dagger(\vec{y})\} = 0\ , \end{aligned}$$

derive the anticommutation relations satisfied by the Fock space operators.

(5) Prove the following identities

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$$\begin{aligned} \text{Tr}\,[\text{odd \# of } \gamma \text{ matrices}] &= 0 \\ \text{Tr}\,[\gamma^{\alpha}\gamma^{\beta}] &= 4g^{\alpha\beta} \\ \gamma^{\mu}\gamma_{\mu} &= 4 \\ \gamma^{\mu}\gamma^{\nu}\gamma_{\mu} &= -2\gamma^{\nu} \end{aligned}$$