QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT II

(1) Derive an expression for $\langle \vec{p}\,|\vec{k}\rangle$. Use this result to motivate the identity operator for the *one-particle states*

$$\mathbb{1} = \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}|\vec{q}\rangle\langle\vec{q}|$$

(2) What is the mode expansion for a non-interacting quantum scalar field $\hat{\phi}(x)$? By considering $\langle\vec{p}\,|\phi(\vec{x})|0\rangle$, interpret the state $\hat{\phi}(x)|0\rangle$.

(3) The *Higgs sector* of the Standard Model of particle physics gives elementary particles their masses. The Lagrangian of this sector has a form very similar to

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4\ .$$

(a) What is the conjugate momentum operator corresponding to ϕ ? What are the canonical commutation relations ?
(b) Corresponding to this Lagrangian density, what is the action, Hamiltonian density and Hamiltonian ?
(c) Use Euler-Lagrange equations to derive the equation of motion.

(4) What is the equations of motion and canonical energy-momentum tensor corresponding to the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ ,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

(5) Let $\Theta_{\mu\nu}$ be the canonical energy-momentum tensor, that we derived in class. Consider a modified tensor

$$\mathcal{T}_{\mu\nu} = \Theta_{\mu\nu} + \partial^\rho\chi_{\rho\mu\nu}$$

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(a) If we choose a tensor such that $\chi_{\lambda\alpha\beta} = -\chi_{\alpha\lambda\beta}$, is the conservation law modified ?

(b) What were the conserved Noether charges for $\Theta_{\mu\nu}$. Do the conserved charges change under this modification ?

This additional freedom may be used to construct a modified symmetric energy-momentum tensor such that $\mathcal{T}_{\mu\nu} = \mathcal{T}_{\nu\mu}$ [J. Belinfante, Physica **6**, 887 (1939)]

(6) What were the Schrödinger and Heisenberg pictures in quantum mechanics ?

Suggested Readings

- "Quantum Field Theory and the Standard Model", Matthew D. Schwartz : Chapters 2, 3.