PHY420 Tutorial 2

Handed out 5.02.2018, Due 12.02.2018

1. Compare the magnitudes of the corrections to the energy given by the independent particle hamiltonian (a) due to the electron repulsion term and (b) due to the spin-orbit coupling term for the 1s2p state of the helium atom.

2. Draw a level diagram after including the $\mathbf{L} \cdot \mathbf{S}$ coupling for the carbon atom in its ground state. Indicate the level splitting (schematic, not to scale) and show the term symbol for each level.

3. What is the difference between the Hartree equation and the Hartree–Fock equation for the $1s^2$ state of helium? Calculate the Hartree potential for this case (the effective potential seen by 1s electron due to the combination of the nucleus and the second 1s electron).

4. Calculate the matrix element for the dipole transition corresponding to the Lyman-α transition.

5. Show that the rate of populating and depopulating a level are complementary in a 2-level atom, under the time-dependent perturbation theory.

6. Using the equation
$$N_1/N_2 = \exp(\hbar\omega_{21}/kT)$$
show that
$$P_{abs} = P_{st.em.} = \langle n \rangle \, P_{spont},$$
where $\langle n \rangle = 1/[\exp(\hbar\omega/kT) - 1]$ is the average number of photons with frequency ω_{21}. Give an interpretation of this formula. When is spontaneous emission dominant? When is stimulated emission dominant?

7. The lifetime of a state is the time it takes for N/e atoms out of a collection of N atoms in the excited state to decay spontaneously to the ground state. Using the formula for the Einstein A-coefficient, calculate the lifetime of the 2p state of atomic Hydrogen. Without explicitly calculating the of the matrix element, explain why the 3s level and the 2s levels are longer lived that the 2p level (by how much, only as order of magnitude?).

8. If we include the first (i.e. the kr) term in the expansion of the external electric field responsible for a radiative transition, we go beyond the dipole approximation. What would be the features of the process that includes this term?