

INELASTIC SCATTERING

(2)

The general form of the (asymptotic) wavefunction in the presence of a scattering centre is given as a sum of incoming and outgoing waves:

$$\psi_{r \rightarrow \infty} = \frac{1}{2ikr} \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) \left[(-)^{l+1} e^{-ikr} + S_l e^{+ikr} \right]$$

as seen before, except that we have replaced $e^{2i\delta_l}$ by S_l . The term S_l represents the amplitude of the outgoing wave (The e^{+ikr} part) and when $S_l = e^{2i\delta_l}$ as we saw earlier for elastic scattering, $|S_l| = 1$

The scattering amplitude and the scattering cross-section is written in terms of the δ_l or rather in terms of S_l for the elastic process as

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) (S_l - 1) P_l(\cos\theta)$$

$$\sigma_{\text{elastic}} = \frac{\pi}{k^2} \sum_0^{\infty} (2l+1) |1 - S_l|^2 \quad \left\{ \equiv \int |f(\theta)|^2 d\Omega \right\}$$

If inelastic processes are present, we cannot take $S_l = e^{2i\delta_l}$ because this choice of $S_l \Rightarrow |S_l| = 1$, which in turn means that the amplitudes of the partial waves are identical (and equal to 1) for the incoming and the outgoing waves. The last part is true for elastic scattering only.

Hence if inelastic processes are present, then $|S_l| < 1$. REFER TO l -partial amplitudes here

The cross-section for inelastic processes must thus be that part that is missing from S_l (ie the part deficient from $|S_l|$ being 1)

Hence

$$\sigma_{\text{inelastic}} = \frac{\pi}{k^2} \sum_0^{\infty} (2l+1) [1 - |S_l|^2]$$

$\sigma_{\text{inelastic}}$ is also called the reaction cross-section.

The total cross-section is

$$\begin{aligned} \sigma_{\text{tot}} &= \sigma_{\text{elastic}} + \sigma_{\text{reaction}} \\ &= \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \left[|1 - S_l|^2 + (1 - |S_l|^2) \right] \end{aligned}$$

$$= \frac{2\pi}{k^2} \sum_0^{\infty} (2l+1) \left[1 - \text{re } S_l \right]$$

If we write the scattering amplitude as in terms of partial waves then $f(\theta) = \sum_{l=0}^{\infty} (2l+1) f_l P_l(\cos\theta)$ where $f_l = \frac{S_l - 1}{2iK}$

The partial wave cross-sections are then $\int |f_l(\theta)|^2 d\Omega$

$$\text{and } \left\| \sigma_{l,el} \right\| = \frac{\pi}{K^2} (2l+1) |1 - S_l|^2 \quad \left\| \sigma_{l,react} \right\| = \frac{\pi}{K^2} (2l+1) (1 - |S_l|^2)$$

$$\left\| \sigma_{tot,l} \right\| = \frac{2\pi}{K^2} (2l+1) (1 - \text{Re } S_l)$$

TRANSITIONS

In \neq inelastic scattering there is a change in the K of the system, this change results in a change in the internal energy of the system. In particular the "target" particle or the "projectile" may undergo internal changes, such as excitation or even fragmentation.

We consider such changes, or transitions, by applying the Fermi Golden rule for transitions in the continuum.

We consider the system going from an initial state $|i\rangle$ to a final state $|f\rangle$, with the condition $E_i = E_f$; E_i and E_f includes all energies of the system. and $|i\rangle$ and $|f\rangle$ are states of the incident particle plus the scatterer. By the Fermi Golden rule the transition probability depends on the matrix element of the transition (squared) and the density of final states

$$\left\| dW_{fi} \right\| = (2\pi/\hbar) |U_{fi}|^2 \delta(E_f - E_i) d\nu_f$$

Specifically, when the incident particle is an electron (or any other particle without a structure, and the scatterer is an atom, then the initial and final states may be specified as $|i\rangle \equiv |\vec{p}, 0\rangle$ and $|f\rangle = |\vec{p}', n\rangle$ where \vec{p} and \vec{p}' are the incident and scattered particle momenta and $|0\rangle$ and $|n\rangle$ refer to the states of the target atom.