Bohr Model Summary

Energy levels are discrete

$$E_n = \frac{me^4}{32\pi^2\hbar^2\epsilon_0^2}\,\frac{1}{n^2}$$

Radii are discrete in the Bohr model

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}\,n^2$$

The radius for $n = 1$ is given the special name, *Bohr Radius*, a_0
Thus

$$r_n = a_0\,n^2$$

Schrödinger Model

Energy levels are the same as in the Bohr model

There is no definite radii, instead there is a distribution of the electron positions for different quantum numbers n, ℓ, m_ℓ

The probability density of the electron cloud is given by

$$P(\vec{r})d^3\vec{r} = |R_{nl}Y_{lm}|^2\, r^2\, d\Omega$$

The radial probability density alone is given by

$$P(r)dr = 4\pi r^2 |R_{nl}|^2\, dr$$

So one can speak of the most probable radius $\tilde{r}$ and the average radius $\langle r \rangle$. It turns out that

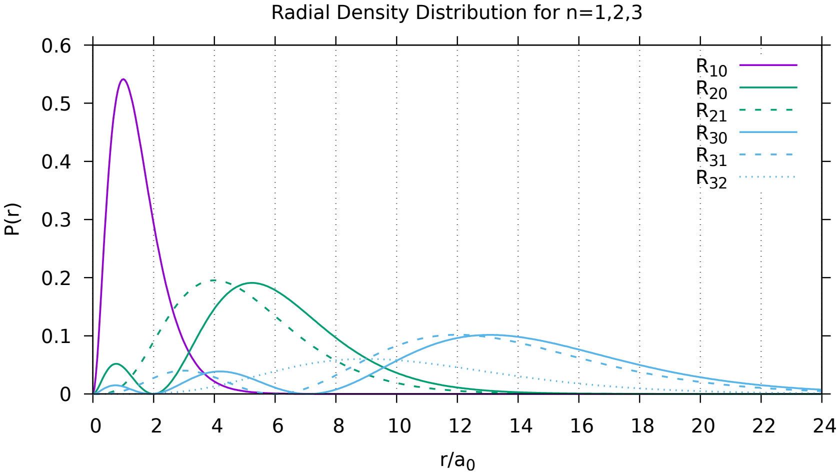
$$\tilde{r}_{n=1} = a_0, \quad \langle r \rangle_{n=1} = 3a_0/2$$

Radial Wavefunctions

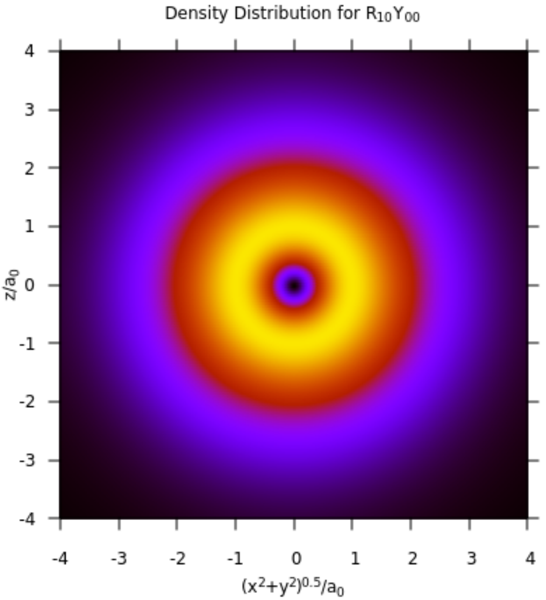
The first few radial wavefunctions are

$$
\begin{aligned}
R_{10}(r) &= \exp(-r) \\
R_{20}(r) &= \exp(-r/2) \times (1 - r/2) \\
R_{21}(r) &= \exp(-r/2) \times (r/2) \\
R_{30}(r) &= \exp(-r/3) \times (1 - 2r/3 + 2r^2/9) \\
R_{31}(r) &= \exp(-r/3) \times (2r/3 - r^2/9) \\
R_{32}(r) &= \exp(-r/3) \times (r/3)^2
\end{aligned}
$$

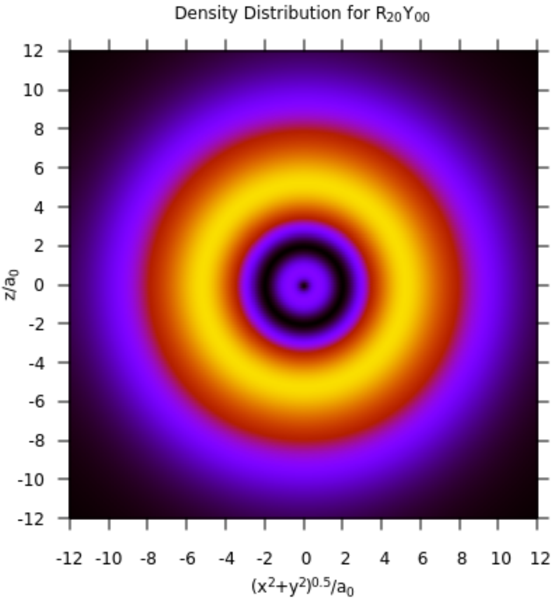
Radial Probability Distributions



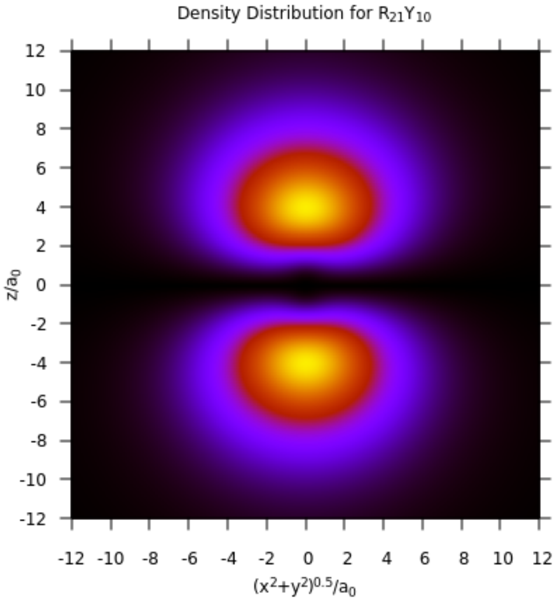
1s probability distribution ($\phi = 0$ plane)



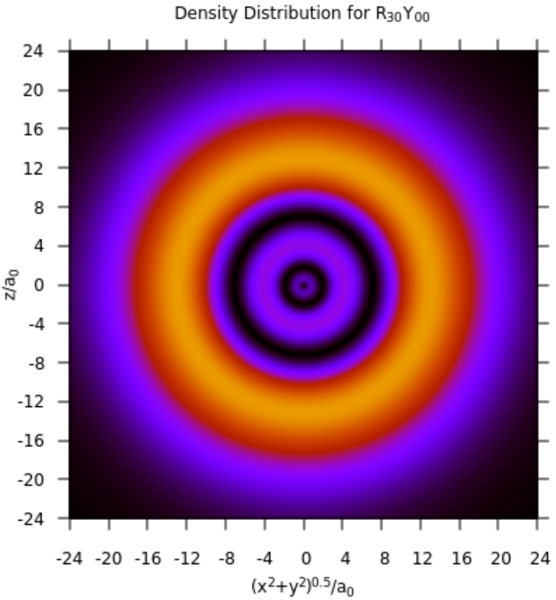
2s probability distribution ($\phi = 0$ plane)



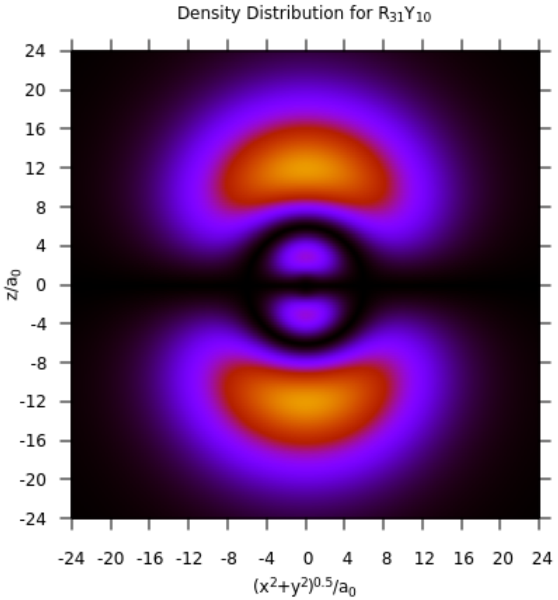
2p probability distribution ($\phi = 0$ plane)



3s probability distribution ($\phi = 0$ plane)



3p probability distribution ($\phi = 0$ plane)



3d probability distribution ($\phi = 0$ plane)