A laboratory experiment on the application of Stefan's law to tungsten filament electric lamps

B. S. N. Prasad[a] and Rita Mascarenhas[b]
Department of Physics, P. G. Centre Mangalagangotri, Konaje, 574152, India

(Received 6 October 1975; revised 24 February 1976)

Commercially available electric lamps are used in a laboratory experiment for the verification of Stefan's law. Assuming that the emissivity of tungsten filament remains constant and that all the filament power goes out as radiation, Stefan's T^4 law can be verified from a log-log plot of radiated power P against temperature T of the filament. It is found that correction to filament power due to convection loss is necessary for the gas-filled lamps and not for vacuum lamps. The experimental value of the slope of the graph of log P vs log T for all these lamps is found to be 5, which shows that the emissivity of the filament surface is proportional to the temperature of the filament. A simplified laboratory experiment avoiding temperature calculations and convection loss corrections is described in this study.

I. INTRODUCTION

Experiments for verifying Stefan's law using electric lamps have been described by others.[1,2] The experiment in its basic form consists of measuring the resistance R of the filament for various values of electric power P dissipated by the lamp. From a knowledge of one reference value of the filament resistance at room temperature (cold resistance), filament temperatures can be computed from a known R-T relation. Assuming that the electric power dissipated by the filament goes out entirely as radiation and that the filament is a perfect black body, the slope of the straight-line graph log P vs log T would give a value of 4 for the exponent in Stefan's law.

Edmonds[1] in his experiment has used a low-wattage electric light bulb in a Wheatstone bridge setup for measuring the filament resistance for various currents and power dissipation and finds that the convection losses are negligible for the bulb used. Wray[2] has repeated this experiment for a low-wattage bulb (6.3 V, 0.1 A) and also a separate experiment using a high-wattage electric lamp (60 W, 240 V, ac) in a simple ammeter-voltmeter setup which includes an autotransformer for voltage variations. From voltage and current readings, filament resistance and power dissipation are known and the filament temperatures are calculated from a linear R-T relation as in Edmonds' method. Wray finds that the convection losses are not negligible for either of the electric bulbs used and from a log-log plot of radiated power against filament temperature the slope of the straight line graph is found to be 4 as in the experiment of Edmonds. However, Wray concludes that this result should not be taken as a successful verification of the T^4 law since the assumption that the emissivity of the filament surface remains constant may not be correct. Contrary to Edmonds' assumption that the surface of the tungsten filament is oxidized during use Wray finds that the filaments of the bulbs used in the experiment are shiny in appearance. Although metal oxides show little variation in emissivity with temperature, the same is not true for tungsten and the emissivity of tungsten varies with temperature.[3] Thus for the very approximate assumption that the emissivity of tungsten filament is proportional to temperature the plot of log P vs log T should give a slope of 5.

We have made a detailed study of this experiment using a number of commercially available electric lamps of different power ratings, vacuum as well as gas filled. We have found that the convection losses are significant for gas-filled lamps only. The filament surfaces of all the lamps remain shiny in appearance after the experiment. In our calculations of filament temperatures we have used the R-T relation which is appropriate for tungsten filament at high temperatures instead of the linear R-T relation used by Edmonds and Wray. From our experiment the slope of the log-log plot of radiated power against filament temperature is found to be 5 for all these lamps.

In this simple experiment for the verification of Stefan's law, the calculation of filament temperature and convection loss corrections are lengthy and tedious. The procedure is simplified by using low-wattage vacuum lamps for which convection corrections are not required and filament temperature calculations are not necessary since a log-log plot of filament current against filament resistance gives the required results.

II. THEORY

Theoretical interpretation of the electrical characteristics of the radiating tungsten filament is based on
(a) the appropriate R-T relation that holds good at high filament temperatures,
(b) the temperature dependence of emissivity of the filament surface,
(c) the corrections to filament input power due to convection losses.

These points are discussed below.

R-*T* relation for tungsten

At high filament temperatures where radiation becomes important, the R-T relation is given[4] by

$$R = R_0 (1 + 5.238t + 0.7t^2 + 0.062t^3) \qquad (1)$$

where $t = (T - 273)/1000$ and R_0 is the resistance at 273 °K.
A simpler formula for calculating the filament temperature is given[5] by

$$R/R_{273} = (T/273)^x, \qquad x = 1.2. \qquad (2)$$

0002-9505/78/4604-0420$00.50

Table I. R/R_0 for different temperatures (°K).T_D, temperature from Dow[6]; T_L, temperature from the linear relation, Eq. (3).

R/R_0	T_D	X	T_L	$(T_L - T_D)/T_D$
10	1990	1.22	1997	0.4%
12	2320	1.21	2375	2.4%
14	2650	1.21	2809	6.0%
16	2960	1.21	3130	6.1%

Table II. Values of cold resistance (R_0) for the electric lamps used in the experiment.

Lamp	R_0 (ohms)
7 W, Vacuum, straight coil	990
25 W, Vaccum, straight coil	220
40 W, Gas filled, coiled coil	100
60 W, Gas filled, coiled coil	83
100 W, Gas filled, coiled coil	34
200 W, Gas filled, coiled coil	21

Also, we have shown in Table I a few representative values of R/R_{300} for various filament temperatures given by Dow[6] which also show that $R \propto T^{1.2}$. In our experiment we have determined the filament resistance at room temperature (~300 °K) very accurately and we have used the graph of R/R_{300} versus T °K prepared from the table of values given by Dow for determining filament temperatures. However, determination of filament temperature is not necessary in the simplified laboratory experiment recommended in this study.

The linear R-T relation used by Edmonds and Wray neglects terms in t^2 and t^3 of Eq. (1), and is given by

$$R = R_0\,[1 + 0.0053\,(T - T_0)]. \qquad (3)$$

As shown in Table I, at high filament temperatures there is a significant difference between the temperatures from $R \propto T$ and $R \propto T^{1.2}$ relation. Filament temperatures from the linear R-T relation reduce the slope of the straight line graph log P vs log T as compared with those from the $R \propto T^{1.2}$ relation.

Emissivity of filament surface

Stefan's law for the rate of energy radiation from a black body of emissivity e, surface area A, at an absolute temperature T is written

$$\frac{dE}{dt} = P = A\,\sigma e T^4, \qquad (4)$$

where σ is Stefan's constant (5.67×10^{-8} W m^{-2} K^{-4}). Assuming that e of the radiating filament is a constant, the slope of the graph of log P vs log T will be 4. The filament surfaces of the electric lamps used in our experiment remain shiny in appearance as in the experiment of Wray which indicates that a clean tungsten surface is exposed. Since the emissivity of tungsten varies with temperature the slope of log P vs log T graph will not be 4. In our experiment we have obtained a value of 5 for the slope which shows that the emissivity of tungsten is approximately proportional to temperature over the range of temperature values encountered in the experiment.

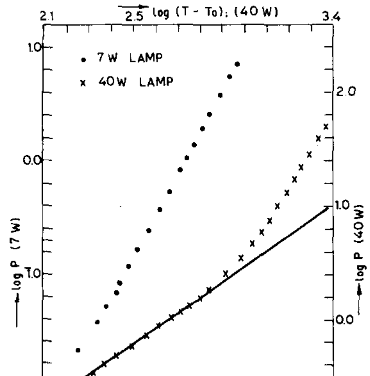

Fig 1. Graphs of log P (total power) against log ($T - T_0$) for 7- and 40-W lamps.

It is possible that depending on the pressure of gas inside the lamp, the residual gas composition, etc., the emissivity of the tungsten filament can be a constant because of oxidation of the filament surface. We have not been able to investigate this aspect of the problem with the commercially available electric lamps. As discussed later we can realize a fictitious perfect black-body radiator in this experiment for which $P \propto T^4$ holds true by using a gas filled lamp and ignoring the convection loss corrections.

Corrections due to convection losses

When both convection and radiation losses are significant the total power loss from the filament is given by

$$\frac{dE}{dt} = X(T - T_0)^n + A\sigma e T^4 - Y T_0^4, \qquad (5)$$

where T_0 is room temperature, T is filament temperature, X and Y are constants. For $T > 1000$ °K, the term in Y can be ignored and for free convection conditions $n = 5/4$. A graph of total filament power against ($T - T_0$) on a log-log scale would yield the value of n in the above formula (see Fig. 1). Also from this graph the power loss due to convection can be noted and hence the radiated power can be found out from the total input power. We have found that the convection losses are significant for the gas-filled lamps only and not for vacuum lamps.

III. EXPERIMENTAL RESULTS

The experimental setup used is similar to the one used by Wray. This arrangement using an autotransformer for voltage variations and ac ammeter and voltmeter for measuring filment current and voltage is well suited for using commercially available incandescant lamps and is preferable to the bridge arrangement used by Edmonds where only low-voltage electric bulbs are used. Also it is easier and more convenient to measure filament resistance from current and voltage readings rather than from bridge measurements. For determining the cold resistance R_0 we find that the extrapolation of the graph of resistance against voltage (or current) to be more reliable rather than the resistance against power. For R_0 determination, it is necessary to take a number of readings at low filament voltages (less than about 5 V). R_0 data for the lamps used in our experiment are given in Table II. However, R_0 determination is

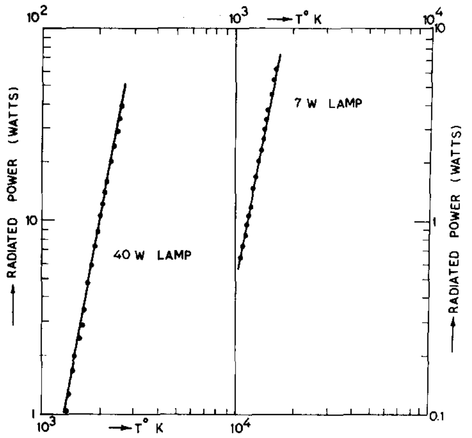

Fig 2. Graphs of P (radiated) against T °K for 7- and 40-W lamps.

not necessary in the simplified laboratory experiment recommended in this study.

We have shown in Fig. 1 representative graphs of log P vs log$(T - T_0)$ for the lamps 7 and 40 W. The graphs for the other vacuum and gas-filled lamps are similar to those in Fig 1. It is evident that the convection losses are negligible for vacuum lamps but not for the gas-filled lamps. The slope of the straight line drawn through the lower points of the graph for the 40-W lamp is 1.3 which indicates the existence of free convection in gas-filled lamps. The power loss due to convection at each of the high-temperature readings is indicated by this graph. Using this and the total filament input power, the power loss due to radiation alone can be determined. We have found that at filament temperatures in excess of 2000 °K, the convection loss is about 20% of the total input power for all the gas-filled lamps. It is interesting to note that by neglecting this correction due to convection loss we get for the exponent in Stefan's law a value of exactly 4 which is 20% less than the correct experimental value of 5 (Fig. 5).

Figure 2 shows the graphs of radiated power against filament temperature for the lamps 7 and 40 W. The slope of these straight-line graphs is 5 which shows that the emissivity of the tungsten filament is approximately proportional to filament temperature and for this radiating body Eq. (4) is of the form $P \propto T^5$. The value of the exponent for the other lamps is also found to be 5.

It is obvious from the above discussion that the calculation part of the experiment is tedious because of the numerous calculations for temperature determination and the need for graphically determining the convection loss. This latter part can be avoided if we use a vacuum lamp but the calculation of resistance, power, and filament temperature are time consuming. This can be minimized to a large extent by following the procedure described below for a vacuum lamp.

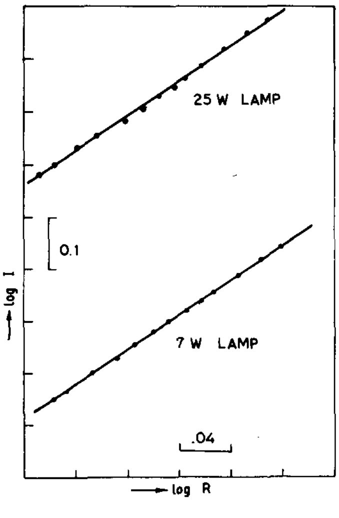

Fig 3. Graphs of log I against log R for the vacuum lamps of 7 and 25 W.

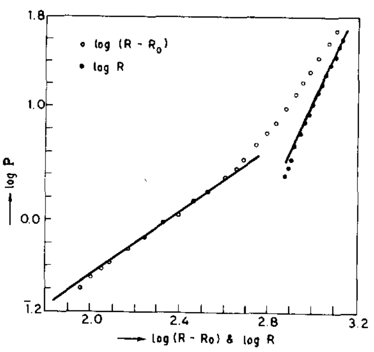

Fig 4. Graphs of log P (total power) against log $(R\text{-}R_0)$ open circles, and log P (radiated) against log R, solid circles, for 60-W lamp.

For a current I in the filament of resistance R and temperature T we write

$$\text{Power} = P = (I^2R) \propto T^N, \quad R \propto T^{1.2}.$$

Hence $I^2 \propto R^{(N/1.2\,-\,1)}$. Thus a graph of log I vs log R would yield a straight line of slope $(N/1.2 - 1)/2$ from which N can be determined. We have shown the graphs of log I vs log R for the vacuum lamps 7 and 25 W in Fig. 3 and from the slopes of these lines we find $N = 5$ which is in agreement with the result from log P vs log T graphs for these lamps.

The above procedure cannot be readily adopted for gas-filled lamps since it is necessary to apply corrections to filament input power due to convection loss. The following method is used.

At low filament temperatures where convection losses are predominant, we can assume the linear R-T relation

$$R = R_0[1 + \alpha(T - T_0)];\ (R - R_0) \propto (T - T_0).$$

Thus from a graph of total filament input power aginst $(R - R_0)$ we can find the power loss due to convection, and determine as before the radiated power at high filament temperatures. Since $R \propto T^{1.2}$ at these high temperatures, a graph of radiated power against resistance of the filament on a log-log scale will yield a straight-line graph of slope $N/1.2$, from which N can be determined. Figure 4 shows these graphs for the 60-W lamp and the value of N is 4.96 which is in good agreement with the value of N from log P vs log T graph.

It is evident that the experiment using gas-filled lamps is more tedious than the one using vacuum lamps even after avoiding calculations of filament temperatures. A simplified laboratory experiment for Stefan's law verification is to use a vacuum lamp and from the slope of the graph of log I vs log R the exponent in Stefan's law can be determined.

The effect of neglecting the correction due to convection loss on the value of the exponent in Stefan's law is seen from the graphs of log I vs log R for the gas-filled lamps shown in Fig 5. From the slope $(N/1.2 - 1)/2$ for these lines we find $N = 4$ for all the lamps. It is likely that one can conclude to have successfully verified Stefan's T^4 law by using

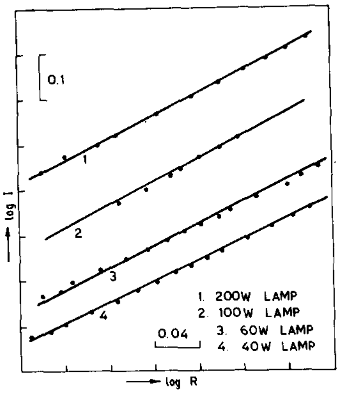

Fig 5. Graphs of log I vs log R for the gas-filled lamps.

a gas-filled lamp in the experiment described above and neglecting convection losses.

The discrepency in the results of a similar experiment by Wray is due to the error in temperature calculations using the linear R-T relation (Table I). With our experimental values we have verified this by determining the slope of log P vs log T using the temperature values from the linear R-T relation and $R \propto T^{1.2}$ law.

IV. CONCLUSIONS

It is seen that a simple laboratory experiment using incandescent lamps for the verification of Stefan's law needs careful interpretation of the experimental results. For the tungsten filament radiator whose surface is oxidized $P \propto T^4$ holds true since metal oxides show little variation in emissivity with temperature. However, the filaments of the incandescent lamps used in our experiment exhibit a clean shiny appearance of a pure metal rather than the dull appearance of oxide coated surfaces and the emissivity of tungsten is temperature dependent. The experimental value of 5 for the exponent in Stefan's law shows that the emissivity of the filament is approximately proportional to the temperature of the filament. From this study it is found that the experimental procedure can be simplified by using a low-wattage vacuum lamp and a simple graph of log I vs log R is sufficient to verify Stefan's law.

ACKNOWLEDGMENT

We are grateful to H. Sanjeevaiah, Reader in Physics, Mysore University for many helpful discussions, to Dr. D. L. R. Setty, Professor and Head of the Physics Department, P. G. Centre, for his keen interest and encouragement in this work. This work is financed by the University of Mysore through grants from the University Grants Commission.

[a] Present address: Teacher Fellow, Department of Physics, Utkal University, Bhubaneswar-751004, India.

[b] Present address: Department of Physics, SUNY at Albany, Albany, New York 12222.

[1] I. R. Edmonds, Am. J. Phys. **36**, 845 (1968).

[2] E. M. Wray, Physics Education (U.K.) **10**, 25 (1975).

[3] *Handbook of Chemistry and Physics* 46th ed., edited by R. C. Weast (Chemical Rubber, Cleveland, 1965), pp. E139, E146, E147.

[4] *International Critical Tables Vol VI*, edited by E. W. Washburn (McGraw-Hill, New York, 1929), pp. 109–120.

[5] J. B. Hoag and S. A. Kroff, *Electron and Nuclear Physics*, 3rd ed. (Von Nostrand, New York, 1948), pp. 118.

[6] W. G. Dow, *Fundamentals of Engineering Electronics* (Wiley, New York, 1952), pp. 572.